

NOTES ON THE FORMATION OF GLAZED FROST.¹

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1. *Introduction.*—The glazed frost is a transparent smooth coating of ice covering trees, or buildings, or the ground, and is usually caused by rain which freezes as it touches these objects and thus covers them with a coating of transparent ice. The formation of this rare phenomenon has been studied by several scientists. Among others, Dr. Meinardus (1) investigated thoroughly the ice storm that occurred over middle and eastern Germany on October 20, 1898, and stated the three conditions necessary for the production of the glazed frost, i. e.:

(1) In the upper air there must be a stratum of warm air having the temperature above 0° C.

(2) This warm layer should be laden with moisture, and must have the vertically upward component of motion so that condensation of vapor takes place in it.

(3) Below this warm layer there must be another layer of the temperature below 0° C.

Dr. Meinardus thinks that raindrops from the upper layer of warm air become undercooled as they fall through the cold air below it. M. Angot (2) states that the glazed frost is caused by raindrops undercooled during their falling through the air and touching the objects, both having the temperature below the freezing point.

The undercooled raindrops will freeze into solid balls of transparent ice when there are some turbulent motions in the layers of cold air through which they fall. As is well known, these frozen raindrops are sometimes called "ice rain" and are quite different from graupeln or soft hails. Hence, we must add the fourth condition to the three separate ones already stated, that the raindrops from the upper warmer air should fall through the layers of comparatively calm air. Recently Dr. G. Hellmann (3) has given an instance of ice rain which was formed by the melted snowflakes falling through the warm layer of air and becoming undercooled in passing through the cold layer below. The question then naturally arises that how the raindrops become undercooled when they fall through the cold air.

Before entering into this inquiry we shall describe a few instances of glazed frosts that have occurred in this country and shall examine the meteorological conditions that then prevailed.

2. *Glazed frost occurred at Tokyo on January 8, 1902.*

On the early morning of January 8, 1902, a remarkable glazed frost occurred at Tokyo and in the neighboring districts. The branches and leaves of trees and all exposed objects were covered with transparent ice of a thickness of about 5 to 10 millimeters. The ground was also covered with a coating of ice to considerable danger to pedestrians. The temperature of air remained 1 degree below the freezing point, but that of the surface of ground was a little above it. The winds blew from the northwest, the force being moderate. Rain fell since 2 a. m. of the day. At about 4 a. m. it turned into sleet. We have no observations from the upper air at Tokyo. But fortunately we have hourly meteorological observations taken on the summit of Mount Tsukuba (870 meters high) at a distance of 65 kilometers to the northeast of Tokyo. From these observations we know that at the height of 870 meters above sea level the strong winds from the southwest were blowing and rain was falling slightly. The temperature of air was above the freezing

point and was about 3 or more degrees higher than that at Tokyo. Hence, we may assume, in all probability, that warm winds from the southwest were also blowing in the upper level at Tokyo, and rain was falling from this warmer air. We give below extracts of the observations made at Tokyo and on Mount Tsukuba.

Tokyo (21 meters).

Element.	Hour.					
	3 a. m.	4 a. m.	5 a. m.	6 a. m.	7 a. m.	8 a. m.
Temperature of air, ° C.	-1.1	-1.0	-1.1	-1.2	-1.2	-1.2
Direction of wind.	N.	NNW.	NW.	NW.	NW.	NW.
Speed of wind, m. p. s.	2.8	4.6	5.4	5.0	3.5	4.3
Relative humidity, %.	83	90	93	95	97	95
Cloudiness.	10	10	10	10	10	10
Rainfall, mm.	0.1	2.8	5.2	4.3	2.7	—
Temperature of ground.	0.1	0.1	0.1	0.1	0.1	0.1

Remarks: ☐° a. p., ☉° 2a—2:30a, 2:45 a—☉, *° 4:10a—☉, *° 6a—6:10a, T°, ~ a.

Tsukuba (870 meters).

Element.	Hour.					
	3 a. m.	4 a. m.	5 a. m.	6 a. m.	7 a. m.	8 a. m.
Temperature of air, ° C.	-1.2	0.7	1.0	2.3	2.8	2.2
Direction of winds.	SW.	SW.	SW.	SW.	SW.	SW.
Speed of winds, m. p. s.	3.9	9.2	10.2	9.8	9.3	12.7
Relative humidity, %.	96	100	100	100	98	95
Cloudiness.	—	—	—	10	—	—
Rainfall, mm.	0.2	0.5	0.7	0.6	0.3	—

Remarks: ☐° a., ☉° 5a., *° 1a., 2:10 a—☉° 3a—7a.

3. *Glazed frost occurred at Asahigawa on March 7, 1914.*—Mr. J. Yamada, director of the meteorological observatory, Asahigawa in Hokkaido, writes me as follows:

On the night of the 6th instant snow began to fall and continued till 1^h 45^m on the morning of the 7th, and then it turned into rain, which continued till 5^h 57^m on the afternoon of the same day. We also experienced a frozen rainfall. The temperature of the air fell below the freezing point since 10 p. m. of the 6th and ranged from -0.8° to -2.2° during the rainy hours.

Glazed frost began to appear at 6 a. m. on the 7th, and became fully developed at about 11 a. m. Thence its formation remained stationary, making no further progress, owing to a slight rise of air temperature. At 5 p. m. the temperature began to fall, but the coating of ice did not increase in thickness, since rain ceased to fall.

In order to give an idea of the thickness of ice on the branches of trees we give the following result of my measurements made at 11 a. m. on the 7th. On a twig of a tree measuring 3 millimeters in diameter the thickness of ice deposited was 2 to 2.2 centimeters on the upper side, and 1.3 centimeters on the lower side of it. The thickness of ice on a telephone wire having the diameter of 2.5 millimeters is 1.2 centimeters on its upper side and 0.5 centimeter on its lower side. The roofs and walls of this observatory were covered with the coating of transparent ice having a thickness of 0.6 centimeter.

During the formation of the glazed frost, strong winds from the north continued to blow, the maximum speed being 7.9 m.p.s. In consequence the glazed frost developed most conspicuously on the north sides of buildings and other substances. But on the branches of trees and telephone wires the ice grew mostly on their upper sides, so that the sections of the ice coating were ellipses, excentric with them. We also observed many icicles hanging from the branches from trees. The diameters of these icicles range from 0.4 to 0.7 centimeter and their length from 3 to 5 centimeters. Owing to the heavy deposit of ice, telephone and telegraph wires in this town were broken in many places. Many electric poles were prostrated. Electric communications in this town were suspended for the whole day. Branches of trees were broken down, and the trunks of larches and poplars were bent down owing to the overloading of the ice coating.

On the afternoon of the 8th the weather cleared up, and the sun shone upon the transparent coating of ice. It remained unmelted till the evening of the 10th.

4. *Formation of glazed frost.*—As we have explained in the first paragraph the point to be explained in the theory

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of the formation of glazed frost is how raindrops become undercooled in passing through a layer of cold air. They may be cooled by conduction and evaporation as it is tacitly assumed in the current theory. Mr. E. Gold (4) calculated the rate of cooling of a drop 2 mm. in diameter and found that the rate is a degree for every meter of fall if the temperature of air is $2^{\circ}\text{C}.$ below that of the drop, neglecting the effect of evaporation. From this we see that with an air temperature below $-2^{\circ}\text{C}.$ the drop would lose sufficient heat in falling through 100 m. to turn it into ice. But as our colleague does not give the method and data of his calculation we can not apply his result directly to the cases under consideration. So, let us newly estimate the rate of cooling of a rain drop taking both conduction and evaporation into account.

Let the difference of temperature of the drop and air be θ , the latter having the uniform temperature. Let the radius of the drop be r , then we have

$$-4\pi r^2 E \theta \, dt - 4\pi r^2 q l \, dt = +\frac{4}{3}\pi r^3 c \rho \, d\theta \quad (1)$$

where E is the coefficient of external conductivity, q the rate of evaporation (per cm^2 of the surface), c the specific heat, ρ the density of the drop, and l the latent heat of vaporization.

We have

$$(E\theta + ql)dt = -\frac{c\rho}{3}r d\theta, \\ -\frac{3}{c\rho r}dt = \frac{d\theta}{E\theta + ql} = \frac{1}{E} \frac{d(E\theta)}{(ql + E\theta)}$$

Integrating we get

$$\theta = \frac{C}{E} e^{-\frac{3E}{c\rho r}t} - \frac{ql}{E}$$

Let the initial temperature of the drop be θ_0 , the origin of the temperature scale being the temperature of the external air. Then

$$\theta_0 = \frac{C}{E} - \frac{ql}{E}$$

therefore

$$\theta = \left(\theta_0 + \frac{ql}{E}\right) e^{-\frac{3E}{c\rho r}t} - \frac{ql}{E} \quad (2)$$

Of the rate of evaporation of raindrops we are quite ignorant. In the annual meeting of the Austrian Meteorological Society for 1896 Dr. W. Trabert (5) read a paper on this subject, but his paper seems to have been published neither in the organ of the society nor in other scientific journals.

In order to get an idea of the magnitude of evaporation we have observed the loss of weight of a water drop which was placed on a thin cover glass coated with paraffin wax. The glass with the drop was placed in a chamber in which the humidity and temperature were kept nearly constant.

We give below the result of our measurements:

Time	Weight of the drop	Radius of the drop
11 h. 19 m. a.	11.7 mg.	1.40 mm.
2 00 p.	6.0 mg.	1.12 mm.
3 26 p.	3.7 mg.	0.96 mm.
4 00 p.	2.9 mg.	0.88 mm.

Since the rate of evaporation of the water drop is proportional to its surface, the radius of the drop decreases

at a constant rate with time. In our measurements this rate is 0.002 mm. per minute or 0.000033 mm. per second. Hence we have

$$q = 3.3 \times 10^{-6} \times 1^2 \text{ mm. per second.}$$

During the experiments the deficiency of saturation of air in the chamber in which the water drop was placed was 3 mm. and there was no sensible motion of air.

Now let us estimate the temperature of raindrops which caused the remarkable glazed frost at Tokyo. Assume that the height of the air stratum from which the rain fell at Tokyo in that case was about 870 meters and the temperature of it was $2^{\circ}\text{C}.$ Since the temperature of air near the surface of the ground at Tokyo was $-1.2^{\circ}\text{C}.$ it is not improbable to assume that the mean temperature of cold air stratum which stretched above the city up to 870 mm. [870 meters?] was $-1.5^{\circ}\text{C}.$ at that time.

Again let us assume that the diameter of the raindrop was 2 mm., and the speed of falling is 6 meters per second. Then the time of falling is $870/6 = 145$ seconds.

The rate of evaporation of water is proportional to the deficiency of saturation of air. In the case under consideration we have assumed that the relative humidity is 90 per cent and the temperature of air is $-1.5^{\circ}\text{C}.$, therefore the deficiency of saturation is only 0.4 mm.

Hence the rate of the water drop will be $q \times \left(\frac{0.4}{3.0}\right)$. But

the rate of evaporation increases proportionally to the square root of the wind velocity. In the actual case the water drop fell with the velocity of 6 meters per second. In the case of the experiments on the evaporation of water drops there was no sensible motion of air, but we may assume that the speed 0.1 meter per second or less.

Hence the rate of the evaporation of the raindrop will be

$$q \times \frac{0.4}{3.0} \times \sqrt{\frac{1+6}{1+0.1}} = q \times \frac{1}{3} = 3.3 \times 10^{-6} \times \frac{1}{3} \\ = 1.1 \times 10^{-6} \text{ mm./sec.}$$

We take therefore

$$t = 145 \text{ sec.} \quad r = 0.1 \text{ cm.} \\ q = 1.1 \times 10^{-6} \text{ mm. p. sec.} \quad l = 600 \text{ gram-calories} \\ \rho = 1 \quad E = 2.66 \times 10^{-4} \\ c = 1 \quad \theta_0 = 2 + 1.5 = 3.5^{\circ}\text{C.}$$

Before calculating the final temperature of the falling drop let us first find the time at which the difference of the temperatures of the drop and surrounding air becomes zero. This difference changes sign at that moment, and then we must change the meaning and value of E .

Let t' be the required time then we have $\theta = 0$ at $t = t'$. Hence we get

$$0 = \left(\theta_0 + \frac{ql}{E}\right) e^{-\frac{3E}{c\rho r}t'} - \frac{ql}{E}$$

From this we have

$$t' = \frac{c\rho r}{3E} \left\{ \log \left(\theta_0 + \frac{ql}{E}\right) - \log \left(\frac{ql}{E}\right) \right\} \quad (3)$$

Now

$$\theta_0 + \frac{ql}{E} = 3.5 + \frac{1.1 \times 10^{-6} \times 600}{2.66 \times 10^{-4}} = 3.5 + 2.5 = 6^{\circ} \\ \frac{ql}{E} = 2.5^{\circ}\text{C.}$$

$$\frac{c\rho r}{3E} = \frac{1 \times 1 \times 0.1}{3 \times 2.66 \times 10^{-4}} = 125.3$$

Hence

$$t' = 125.3 \times (1.79 - 0.92) = 125.3 \times 0.87 = 109 \text{ seconds.}$$

In finding the final temperature of the rain drop we introduce a new quantity E' instead of E and put θ_0 equal to zero in the equation giving the value of θ . Then we get

$$\theta = \frac{ql}{E'} \left(1 - e^{\frac{3E'}{E'} t''} \right)$$

in which we shall put $145 - 109 = 36$ for t'' . In actual calculation we put E equals to E' in absolute values, since we have, for the present, no adequate value for E' .

Then we have

$$\theta = 2.5 \times \left(1 - e^{\frac{36}{125.3}} \right)$$

$$= 2.5 \times (1 - 1.33) = -2.5 \times 0.33 = -0.8^\circ \text{C.}$$

This is of course the same result which we obtain by putting $t = 145$ in equation (2).

Hence the final temperature of the drop will be

$$-0.8 - 1.5 = -2.3^\circ \text{C.}$$

From the above calculation we see that in the case under consideration the conduction and evaporation of rain drops falling through ice-cold layers of the atmosphere will be sufficient to cool them many degrees below the freezing point, and to cause them to cover the objects with the coating of ice when they come to touch these objects. In some cases in which the air is too moist the drops of rain cool to the dew point of the air after they fall a few meters from their mother clouds, and condensation begins instead of evaporation on their surfaces. Hence the formation of glazed frost is not probable in such cases.

In conclusion I express my sincere thanks to Drs. K. Nakamura and S. Fujiwhara of the Central Meteorological Observatory for their kind suggestions in the preparation of this note, and also to Mr. J. Yamada of the meteorological observatory, Asahigawa for his kindness in sending me the report of the remarkable glazed frost given in the text.

REFERENCES.

- (1) Meinardus, W. Der Eisregen vom 20. Oktober 1898 über Mittel- und Ost-Deutschland. Wetter, Berlin, 1898, p. 253.
- (2) Angot, A. Traité élémentaire de météorologie. Paris, 1899, p. 257.
- (3) Hellmann, G. Ueber die Entstehung von Eisregen. Bericht. d. K. preuss. Akad. Wiss., Berlin, 1912, p. 1048.
- (4) Gold, E. Glazed frost. Symons's met. mag., London, 1914, 48, p. 233.
- (5) Trabert, W. Verdampfung von Regentropfen. Meteorol. Ztschr., Wien, 1896, 13. Jhrg., p. 323.

HAZE OF MAY 13 TO MAY 17, 1914.

From the 13th to the 17th of May, in that portion of the United States extending from Virginia north to Maine and west to South Dakota, numerous observers reported the presence of haze which, on investigation, proves to have been principally of a reddish brown tint, apparently due to the dissemination of dust from forest fires. It does not seem to have been due to any special distant volcanic eruption as imagined by some. At Mount Weather, Va., the haze was very dense on and after May 19, but Prof. H. H. Kimball at that place reports that he finds nothing of special meteorological character that can be spoken of as important.—[C. A.]

THE THERMAL REGIONS OF THE GLOBE.¹

By A. J. HERBERTSON, M. A., Ph. D., Professor of Geography, Oxford University.

In the paper on the thermal regions of the world, of which I propose to present an abstract, there are a number of sections about which I shall say nothing or next to nothing—e. g., the history of isothermal maps and the description of the different thermal regions. The reason for bringing the subject before this [Research] Department is to discuss the different methods in which the thermal conditions of the earth's surface can best be represented for the use of geographers, and the different attempts I have made to do this, which I wish criticized. In what follows I assume the temperature data are the most reliable available, and that the ordinary precautions have been taken in dealing with them.

If we start with the ordinary isothermal maps of the world, we have in Dr. Buchan's maps in the "Challenger Report" the one consistent set of maps which shows the distribution of temperature at sea level for every month of the year. There have been newer maps showing the mean annual temperature for July and January, but at the present time it would take many months to make maps more satisfactory than the *Challenger* ones for the different months of the year.²

The ordinary isothermal map contains many lines which, from the point of view of the study of the physics of the atmosphere, are desirable and essential; but for most geographical purposes it is necessary to choose from them those which have the greatest geographical significance. For simplicity's sake I have chosen to deal at present with the lines of 0° , 10° , and 20° C. (32° , 50° , and 68° F.). I have chosen 10° (50° F.) because that line roughly represents the average temperature for the month during which growth becomes active for most plants of economic importance in temperate regions. I have chosen the 20° (68° F.) [line] because it marks roughly on the world the boundary between regions where subtropical products can mature and those where they can not.

Taking those three isothermal lines it is possible from the ordinary isothermal map showing the mean annual temperature to make a map which shows temperature belts, but no map which merely shows the annual temperature is satisfactory. You are all familiar with the objections which are obvious on examining figure 1.³ On the west coast in our latitudes the mean annual temperature is the mean of temperatures which differ very little from it, whereas on the east coast the same mean annual temperature is found in a region with very much colder winters and hotter summers; consequently, some seasonal isothermal line must be introduced in any maps which show the thermal zones.

In figure 2 is a reproduction⁴ of Dr. Supan's map in which he combines mean annual isotherms and mean monthly isotherms taking the mean annual temperature of 20° C for the coldest month. I also have published maps showing the thermal zones, using the lines of 0° , 10° , 20° C., for the warmest and for the coldest month (fig. 3). This map is made by superposing the January and July isothermal maps showing the isotherms of 0° , 10° , 20° C. reduced to sea level.

¹ Abstract of a paper presented at a meeting of the Research Department, Dec. 14, 1911.

² Reprinted from The Geographical Journal, London, November, 1912, 40, p. 518-532.

³ See Bartholomew's Atlas: Meteorology, Plate 3.—[C. A., Jr.]

⁴ This and other figures of the original published paper are not reproduced here. The maps referred to as "figure 1" and "figure 2" are to be found in Bartholomew's Physical Atlas: Meteorology, Plate 1, Inserts "Mean annual temperature" and "Temperature zones," respectively.—[C. A., Jr.]